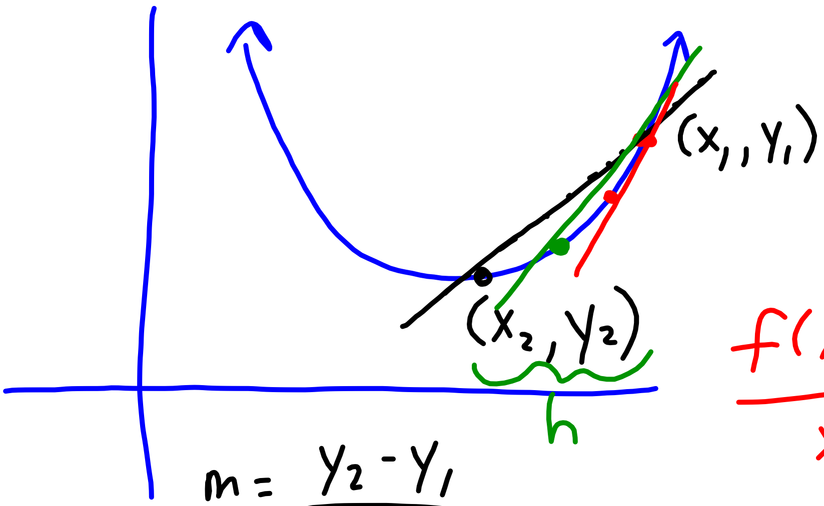


$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Sep 23-8:49 PM

Definion of Derivave

The derivave of a funcon f(x) with respect to the variable x is the funcon f'(x) whose value at x is given by:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

f prime of x

$\cancel{x} + h - \cancel{x}$

if the limit exists.

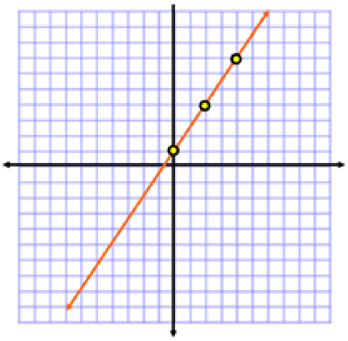
$(x, f(x))$

$(x+h, f(x+h))$

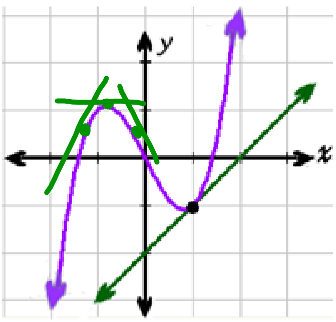
Derivative ⇄ Slope ⇄ Rate of Change

The only difference between the derivative in Calculus and the slope in Algebra is that in Algebra the slope is fixed at a constant value everywhere in the function whereas in Calculus the slope is ever changing throughout the domain of the function.

Algebra



Calculus



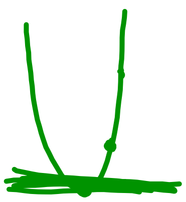
Sep 18-11:17 AM

Ex1. Use the definition to find the value of the derivative

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

1.) $f(x) = x^2$

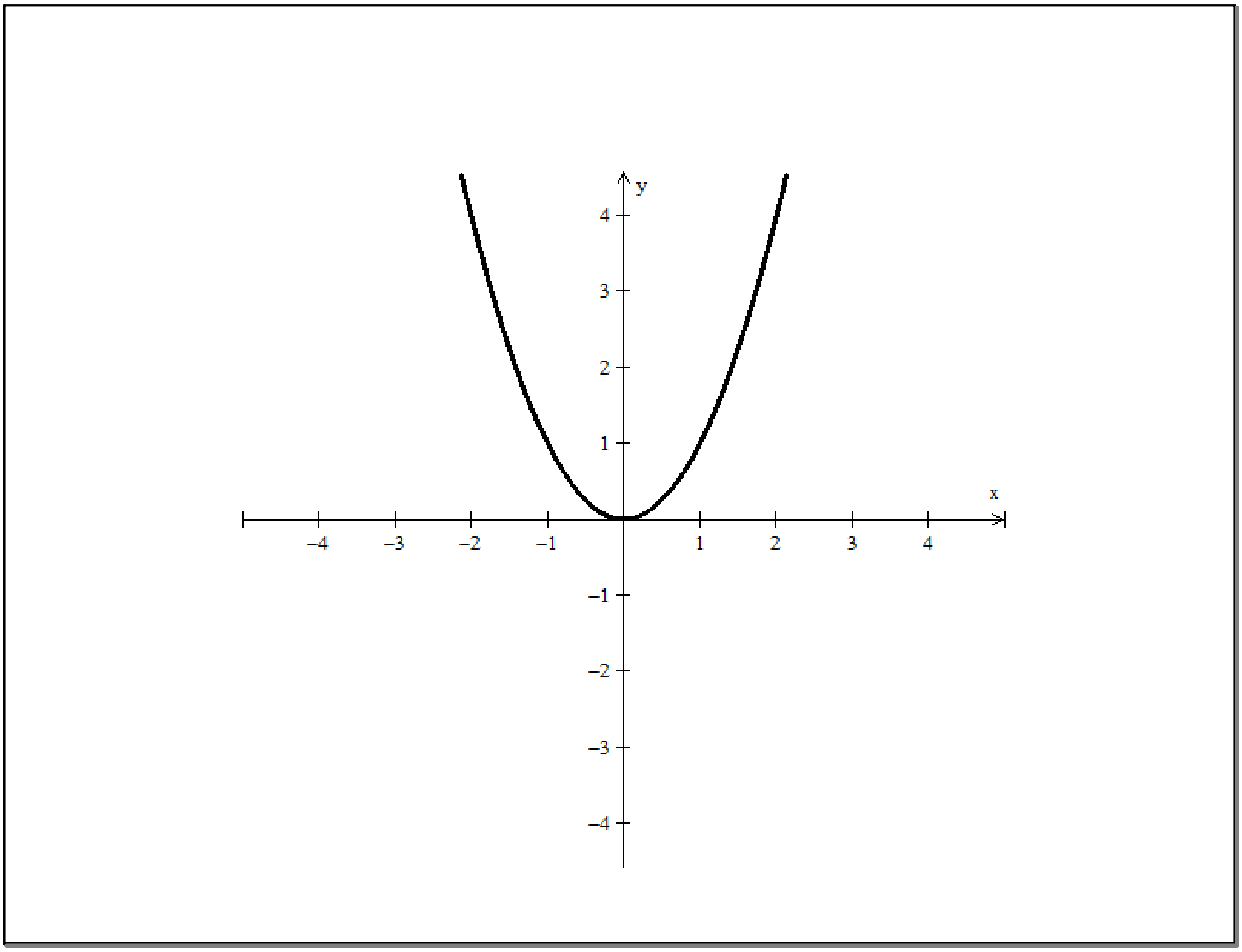
$f(x+h) = (x+h)^2$



$$\lim_{h\to 0} \frac{\overset{(x+h)(x+h)}{(x+h)^2} - x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\overset{h(2x+h)}{2xh+h^2}}{h} = \lim_{h\to 0} 2x+h = 2x$$

$f'(x) = 2x$



Sep 23-9:09 PM

2.) $g(x) = 6x^3$

$(x+h)(x+h)(x+h)$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{6(x+h)^3 - 6x^3}{h}$$

$$= \lim_{h \to 0} \frac{6(x^3 + 3x^2h + 3xh^2 + h^3) - 6x^3}{h} = \lim_{h \to 0} \frac{\cancel{6x^3} + 18x^2h + 18xh^2 + 6h^3 - \cancel{6x^3}}{h}$$

$$= \lim_{h \to 0} \frac{h(18x^2 + 18xh + 6h^2)}{h} = \lim_{h \to 0} 18x^2 + 18xh + 6h^2 = 18x^2$$

$$(x+h)(x+h) = (x^2 + 2xh + h^2)(x+h)$$

$$= x^3 + 3x^2h + 3xh^2 + h^3$$

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3.) $\underset{f(x)}{y} = x^3 + 4x^2$ $\quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$\lim_{h \to 0} \frac{(x+h)^3 + 4(x+h)^2 - (x^3 + 4x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{4x^2} + 8xh + 4h^2 - \cancel{x^3} - \cancel{4x^2}}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 + 8x + 4h = 3x^2 + 8x$$

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Notation

$y =$

$y' =$

y prime

$f(x) =$

$f'(x) =$

f prime of x

$y =$

$\frac{dy}{dx} =$

The derivative of y with respect to x

Ex2. Write the equation of the tangent line to $f(x)=x^3+4x^2$ at x=2. $(2,24)$

$f'(x) = 3x^2+8x$ $m=28$

$28(x-2) = y-24$

$\underline{x=-1}$ $(-1, 3)$ $-5(x+1) = y-3$

$m=-5$

Sep 18-11:17 AM

Show that the derivative of

$f(x) = \frac{3}{x}$ is $f'(x) = \frac{-3}{x^2}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{3x}{(x+h)x} - \frac{3(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{3x}-\cancel{3x}-3h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-3h}{x(x+h)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-3h}{xh(x+h)}$$

$$= \lim_{h\to 0} \frac{-3}{x(x+h)} = \frac{-3}{x^2}$$

Oct 2-10:10 AM

Ex3. The average monthly temperature for Minneapolis, MN is given in the table below.

Month	Temp (F)
January	11.8
February	17.9
March	31.0
April	46.4
May	58.5
June	68.2
July	73.6
August	70.5
September	60.5
October	48.8
November	33.2
December	17.9

a.) Make a scatter plot of this data on your graphing calculator b.) Estimate the derivative of the function for each month by approximating the rate of change of the temperature.

Month (x)	Temp (F) (y)	Derivative
January 1	11.8	-6.1
February 2	17.9	6.1
March 3	31.0	13.1
April 4	46.4	15.4
May 5	58.5	12.1
June 6	68.2	9.7
July 7	73.6	5.4
August 8	70.5	-3.1
September 9	60.5	-10
October 10	48.8	-11.7
November 11	33.2	-15.6
December 12	17.9	-15.3

(1, 11.8)
(2, 17.9)

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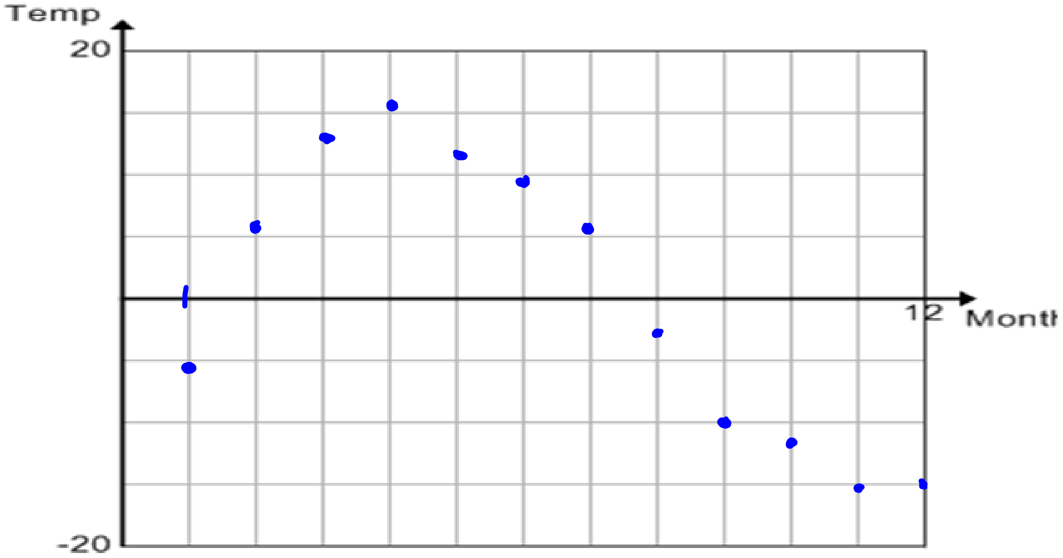
c.) Sketch a graph of the derivative for each month with pencil and paper.



d.) What is the meaning of the derivative in the context of this problem?

e.) What is the units of the derivative?

f.) When the derivative is positive, what does that mean? When the derivative is negative, what does that mean?

g.) In what month(s) is the temperature changing the most rapidly?

h.) In what month(s) is the temperature changing the least rapidly?

Homework

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